Parametric Skin Design Method Based on Plane Crystallographic Group Operation Principle

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Abstract. Under the dual constraints of industrialization and digitalization, the building skin and structure are further integrated to form standardized units to meet the requirements of architectural performance, industrial prefabrication and "complexity" aesthetic characteristics. The complex and diverse forms of today's building skin hide profound mathematical logic relations and operation rules of form generation. Crystallographic group with regular symmetry and the operation principles reflected by it is one of the most important rules and methods of form and pattern processing in skin design. The study of the mural symbols in ancient Egypt, the murals in the Alhambra, the manuscripts of Escher and the window lattice in ancient Chinese architecture profoundly reflects the basic operation principle of crystal group in shaping the skin form of architecture. Abundant and diverse architectural skin forms can be formed through the operation of symmetry group on basic graphic units. On the basis of clarifying the basic principle of crystal group action, the operation matrix of crystallographic symmetry group can be transformed into parameterized operation steps through programming language for visual operation, and then the skin form with high complexity and leap dimension can be generated by geometric algorithm, and the design method of building skin generation based on crystallographic group is constructed. In the selection of operation form, combined with the calculation of building performance and structure, the construction skin can be used in practical engineering is generated. Based on crystallographic group operation, the unifications of building skin and the classification simplification of components can meet the requirements of modular and unifications design in the process of building industrialization, and meet the requirements of current building industrialization and digitization. It has great research significance and value in the aspects of design and construction efficiency and material economic cost.

Keywords: Parametric skin · Crystallographic group · Design method · Operation principle

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1 Introduction

At present, the architectural skin design tends to use parametric design tools to express its external artistry guided by the expression of complexity aesthetics under the operation of flat two-dimensional level. Under the development trend of building skin, the geometric pattern of skin structure design has developed from simple and orderly in the early stage to the nonlinear expression of composition and tends to pursue the nonlinear aesthetic characteristics of generating order by internal mathematical logic. In various geometric and mathematical models, this paper discusses the operation of translational symmetry based on crystal group in skin design.

The theory of translational symmetry was basically developed and logically perfected in crystallography. Symmetry is a very intuitive concept, which can be seen everywhere, such as plants, animals, human beings and natural minerals all have symmetry. Long before humans had the knowledge of group theory, many civilizations realized that two-dimensional crystals had only 17 symmetries, which was reflected in the field of architecture. Ancient Egyptian mural symbols, the murals in the Alhambra Palace, escher's manuscripts, and the window lattice in Chinese classical garden architecture were all great creations based on human cognition of the aesthetic sense of translational symmetry. The creation of murals and window mullions has some similarities with the architectural skin design in essence. In this paper, the parametric skin design method based on the principle of crystal group operation is studied. The 17 plane crystal groups of crystals are applied to the design of building skin to achieve the purpose of intensive cost efficiency of industrial production. At the same time, the nonlinear change supported by the logic of crystal group operation is pursued with the help of parametric design tools combined with algorithms. This paper breaks through the limitation of the design of the preformed skin under the crystal group operation logic and performs nonlinear operation on the preformed skin under the crystal group operation logic.

Based on crystal group design, building skin unit meets the requirements of modular and unitized design in the process of building industrialization and has greater research significance and value than the existing skin design research in terms of design and construction efficiency and material economic cost. Matrix can be obtained by the crystal group operation computer method for generating algorithm, through the generation algorithm based on crystal group operating principle and geometric algorithms of building skin design method can be in the future to create efficient economic and rich with nonlinear change of epidermis forms, with the research and application of high potential, can provide some enlightenment to the construction epidermis design of the future.

2 Algebraic Basis and Principle of Crystallographic Symmetry Group

Translational symmetry can be represented by the mathematical concept of group. Its research has been developed in the field of crystallography, and with it, it brings new design methods to architecture and art design in the process of digitalization. Plane space group is also known as "wallpaper group". Long before the group theory existed, the 17 symmetries of two-dimensional crystals had been widely applied in the field of

architectural decoration, as reflected in the window lattice design of ancient Chinese buildings. With the development of industrialization and digitization of architecture, it is also suitable for building skin design. The translational symmetric operation matrix can provide operability for the parametric design of building skin through computer programming and form the parametric design process of building skin.

2.1 Crystallographic Plane Group and Symmetry Operation

Crystals in nature are composed of atoms, or groups of atoms, arranged regularly in three dimensions, and thus have a regular shape. The minimum repeating unit cells of the crystal are densely packed in three dimensional space, that is, the crystal has translational symmetry. Reaction in a graph, that is, the graph is composed of two or more parts, after a certain linear transformation, the whole graph remains unchanged after the transposition of each part.

Translation symmetry can be represented by the mathematical concept of group. Translational symmetry limits the crystal repetition element to only n = 1, 2, 3, 4, 6 rotation axes, namely the crystallography constraint theorem. The symmetry of the monocell limits the crystal to only 32 point groups. The combination of 32 point groups and translation operations in three-dimensional space determines that the crystal has only 230 space groups. In the two-dimensional case, n = 1, 2, 3, 4, and 6 rotation axes can be intuitively understood from the fact that only square, rectangle, regular triangle, and regular hexagon can be repeatedly filled with plane space, while 5-sided and N (>6) sided cannot be filled with plane space. Therefore, only 10 point groups can be obtained by adding mirror reflection. Only 17 kinds of two-dimensional space groups can be obtained by combining 10 kinds of point groups with translation operations in two-dimensional space. Operations that keep the whole figure unchanged are called symmetry operations, that is, operations in which the distance between any two points of the object remains the same before and after the operation. Symmetrical operation is divided into point symmetrical operation and non-point operation. Point symmetry operation has at least one point in the space does not move during operation, including identical operation, rotation, inversion, mirror reflection and rotation inversion (Table 1). Non - point operation includes spiral rotation and slip reflection. The matrix equation of point symmetric operation is expressed as:

$$\begin{bmatrix} \widetilde{x} \\ \widetilde{y} \\ \widetilde{z} \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{1}$$

Abbreviated to:

$$\widetilde{x} = Wx \tag{2}$$

In the two-dimensional plane of crystallography, there are 17 kinds of plane space groups with 10 kinds of plane point groups and 5 kinds of plane lattice combinations. The plane point group is a combination of all symmetrical elements at one point, and the introduction of 10 plane point groups into the parametric design of building skin

Table 1. 5 Point symmetry operations

Type	Meaning	Graphic representation	matrix representation
Identity operation	No operation is performed		$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Rotation	It's rotating 2 PI over n angles about some axis (N is the rotation axis, pure rotation)		$C_{nz} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Inversion	To change the right hand into the left by a central inversion Changed the right—hand orientation of the image		$i = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
Reflection	A reflection of a plane. Change the right—hand orientation of the graph		$\alpha = \begin{bmatrix} \cos 2\beta & \sin 2\beta & 0 \\ \sin 2\beta & -\cos 2\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Rotation and inversion	Compound operation The product of two operations, rotation and inversion		$S_n(z) = \begin{bmatrix} \cos\frac{2\pi}{n} & -\sin\frac{2\pi}{n} & 0 \\ \sin\frac{2\pi}{n} & \cos\frac{2\pi}{n} & 0 \\ 0 & 0 & -1 \end{bmatrix}$

can create a variety of building skin unit forms based on rotation and mirror reflection operation. Planar lattice represents the spatial lattice form of atoms arranged regularly in crystals, which is applied to the design of building skin and fits with the structural units of building skin, and is used to support and arrange and combine the skin filling units formed based on point group operation. The plane space group of plane point group and plane lattice is arranged and combined based on translational symmetry,

which reflects that 17 kinds of permutations and combinations of skin unit types can be applied to form a complete building skin. The common characteristics of contemporary architectural skin design and two-dimensional space group research determine that if the minimum repeating unit of architectural skin is created through point group operation principle, and the space group operation principle corresponding to repeating unit is selected based on planar lattice, complete architectural skin prototype can be generated based on translational symmetry operation. The inclusion of late interference algorithms can create rich skin forms that adapt to both industrial and digital architectural design.

2.2 Crystallographic Plane Group and Architecture Skin Design

From crystal point group to lattice group and then to space group, it shows the characteristics which are consistent with the parametric design of architectural skin in the aspect of architectural industrialization. Simple to understand, the architect can select 5 kinds of lattice in the form of epidermal structure unit, and then design the corresponding lattice skin filling unit based on point group operating in the form of form, 10 kinds of planar point group internal repeating unit design combining with 5 kinds of planar lattice after some permutation and combination can be created based on 17 kinds of planar space group operating principle of the construction epidermis. The skin structural units based on planar lattice form have the possibility of translational symmetry operation on the building surface, which can form a complete parametric building skin with certain symmetry based on mathematical model. This series of operations can be realized by transforming the planar symmetric group operation matrix of crystallography into a complete parametric skin design process through computer programming language, and transforming the abstract mathematical model into a visual parametric skin design operation (Fig. 1).

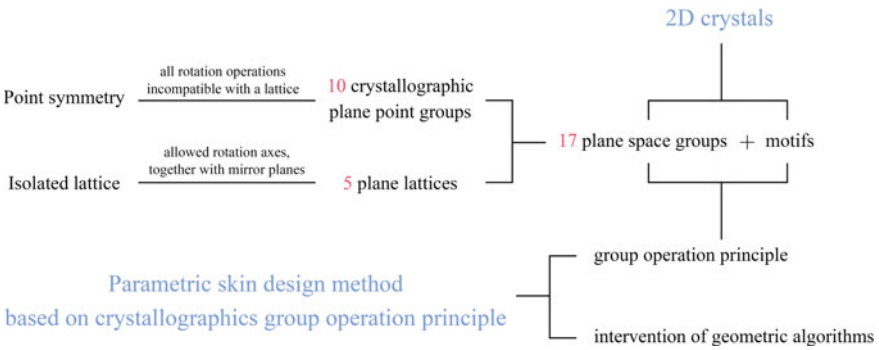


Fig. 1. Plane group operation principle intervention skin design schematic

3 Building Skin Form Based on Crystallographic Group Operation Principle

The parametric design method of building skin based on the operation principle of plane space group makes it possible to break the orderly order of architectural design under the restriction of the established geometric system of architectural design module system. As the key design research, translational symmetry of the unit design for operation, not only emphasizes the design unit in the repeat, also stressed that the skin cell and the overall relationship between the organization, is one of the most critical operation process contains profound mathematical logic and geometric relations, it also provides a precondition for the implementation of parameterized. The geometric transformation relationship of group operation is closely combined with building structure and building materials.

The architectural skin can be regarded as a two-dimensional plane formed by the arrangement and combination of elements, which has certain commonness and connection with the arrangement and combination of two-dimensional crystals, which indicates the feasibility of the translational symmetry operation principle of planar crystal group in the field of skin design. The architect can use 5 planar lattices as the operation base of the skin unit for parametric creation, and select the combination principle of symmetry operation such as rotation and reflection of 10 planar point groups to create a variety of skin unit pattern types for the building skin. The operation principle of translational symmetry of 17 kinds of plane space groups is taken as the parametric transformation and generation algorithm, and the unit patterns are subjected to the orderly repeated symmetrical operation in two-dimensional space. The architectural skin created is not only diversified in pattern, but also follows the internal mathematical logic of parametric design, in line with the development trend of building digitization and industrialization.

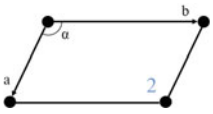
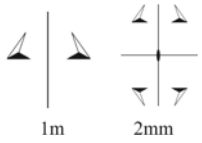
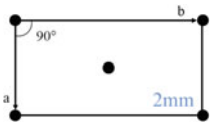
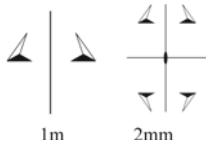
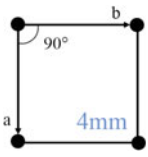
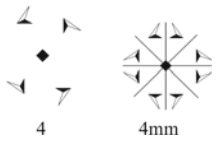
3.1 Plane Point Group and Building Skin Unit

After a series of translational symmetry operations, the plane point groups are arranged and combined on the lattice, and it can be concluded that there are altogether 17 space groups in a two-dimensional plane. From the perspective of operationalism, the architectural skin units generated based on the rotation of the symmetrical elements of the plane point group and the mirror reflecting the operation principle are characterized by rich and orderly patterns, limited types of units and convenience for industrial production and assembly of buildings. Then the complete skin pattern can be obtained by combining with the operation principle of plane space group of corresponding lattice translational symmetry on the plane.

There are 5 lattice types in two-dimensional crystal, which are presented as lattice shapes of Oblique, Rectangular, Square and Hexagonal, corresponding to 10 planar point group operations respectively (Table 2). In the parametric design of skin, if the

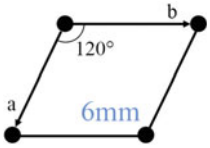
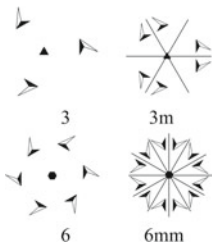
skin filling pattern is the symmetrical element, then the lattice pattern is the structural unit for industrial prefabrication. If the structural elements are generated as symmetric elements, the lattice form becomes virtual body, and the structural elements suitable for various skin inlays can be generated. Lattice is the carrier of point group operation. Only plane point group under lattice constraint can form 17 plane space groups through translational symmetry operation. Therefore, whether the filling pattern of the skin or the structural rod of the skin is used as a symmetrical element for point group operation, the tightly laid complete building skin can be obtained by virtue of lattice constraints. The following table uses a symmetrical pattern as an example to enumerate architectural skin unit patterns that can be obtained by refining the symmetry operation principle of crystallographic plane point groups.

Table 2. 7 plane lattices and 10 plane point groups

Lattice type	Lattice symbol	Lattice diagram	Lattice parameters	Point groups	Point groups diagram (Case of skin unit)
Oblique	mp		$a \neq b$ $\angle\alpha \neq 90°$	1 2	
Rectangular primitive	op		$a \neq b$ $\angle\alpha = 90°$	1 m 2 mm	
Rectangular centred	oc		$a \neq b$ $\angle\alpha = 90°$	1 m 2 mm	
Square	tp		$a = b$ $\angle\alpha = 90°$	4 4 m	

(*continued*)

Table 2. (*continued*)

Lattice type	Lattice symbol	Lattice diagram	Lattice parameters	Point groups	Point groups diagram (Case of skin unit)
Hexagonal	hp		a = b ∠α = 120°	3 3 m 6 6 mm	

3.2 Plane Space Group and Building Skin Form

At present, patterns are increasingly becoming the prototype of nonlinear composition of building skin. The skin units generated by plane group principle can not only correspond to the skin Mosaic units, but also correspond to the structural units in the building skin to form the building skin with Mosaic characteristics. In this way, group principle becomes a mathematical logic and operation method to control and generate building skin. In transforming patterns into skins, in addition to the selection of repeating patterns themselves, the selection of lattice forms and the type of point group operation are also related to the physical properties of the building. Only when these elements are combined with the symmetrical pattern of crystal group operation, can the group operation of two-dimensional pattern be connected with the epidermal material, connection mode and function, bringing richer differentiation and design possibilities. Two-dimensional is not absolute two-dimensional. The combination of planar crystal group operation and planar concave and convex function can realize the jump dimension operation, which is more conducive to the physical performance of the building. The lattice types and rotation, translation and reflection operations involved in the 17 space group operations are shown in the table below (Table 3).

After comprehensive design of various design elements, firstly, lattice types for parametric operation are determined, and corresponding point group forms are selected. Then, repeated patterns are designed according to the principle of symmetric operation at different points to obtain the filling pattern units of the building skin. Finally, translational symmetry is carried out to obtain the laid out building skin. This skin design process based on the operation principle of plane space group can generate various forms of building skin prototype, providing objects for the next step of geometric algorithm intervention. The following figure illustrates the principle of skin pattern generation based on 17 kinds of planar space group operation by taking a symmetrical element as an example. Designers can create rich artistic effects of architectural appearance by designing symmetrical elements and combining architectural skin design elements (Fig. 2).

The interior tiles of The Stella Retail store in Manhattan are a typical example of using the p6 spatial group operation principle to generate a design with architectural

Table 3. P17 plane space groups

Symmetry group	IUC notation	Lattice type	Rotating degree	Reflection axis
1	p1	Oblique	With no rotation	No reflection
2	p2	Oblique	180° rotating	No reflection
3	pm	Rectangle	With no rotation	Parallel
4	pg	Rectangle	With no rotation	No reflection
5	cm	Rhombus	With no rotation	Parallel
6	pmm	Rectangle	180° rotating	90° apart
7	pmg	Rectangle	180° rotating	Parallel
8	pgg	Rectangle	180° rotating	No reflection
9	cmm	Rhombus	180° rotating	90° apart
10	p4	Square	90° rotating	No reflection
11	p4m	Square	90° rotating^{+}	45° apart
12	p4g	Square	90° rotating*	90° apart
13	p3	Hexagonal	120° rotating	No reflection
14	p31m	Hexagonal	120° rotating*	60° apart
15	p3m1	Hexagonal	120° rotating^{+}	30° apart
16	p6	Hexagonal	60° rotating	No reflection
17	p6m	Hexagonal	60° rotating	30° apart

$^{+}$:All centers of rotation are located on the reflection axis
*: Not all centers of rotation are located on the reflection axis

requirements. Each Hexagonal tile is a repeating unit, which is the point group unit of the Hexagonal lattice generated by the Hexagonal rotation of petals as a symmetric element of point group. In accordance with the external artistic requirements, the thickness of each unit is concave and convex, and the symmetrical elements are convex, so that the raised point group pattern and the hexagon form an inclined curve in the honeycomb structure. The glass unit of Trutec Building in Seoul is also a Rectangle lattice stacked operation. The glass in the lattice unit is combined with the pattern elements to make different angles, refraction produces a kaleidoscope of light and shadow changes. The above cases show that the parametric design method combining skin design elements with planar crystal group operation principle can produce rich architectural skin effects (Fig. 3).

The above diagram illustrates the operation principle of skin pattern of 17 planar space groups through symmetry operation of single repeating element. Now that the skin filling pattern as action object can get rich form of epidermis, if the structure of the skin bar as a symmetric operation of symmetry elements for operation, through the computer bar gives the rods plane symmetry group symmetry matrix operation, set the boundary conditions, can make its evolution from a single cell iteration to spread the whole building skin. Many kinds of complete building skin forms can be obtained by

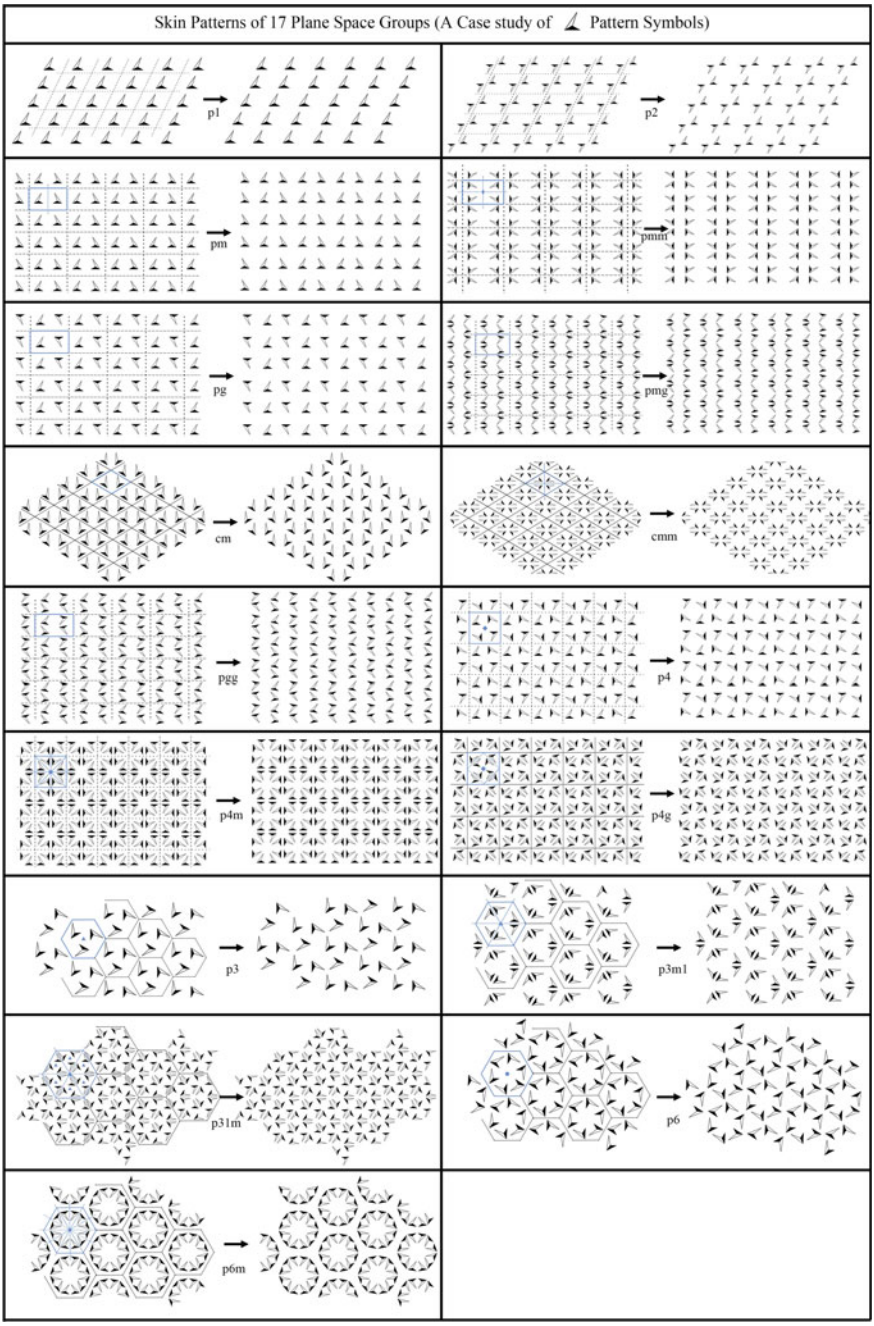


Fig. 2. Skin pattern generation principle for 17 plane space group operations

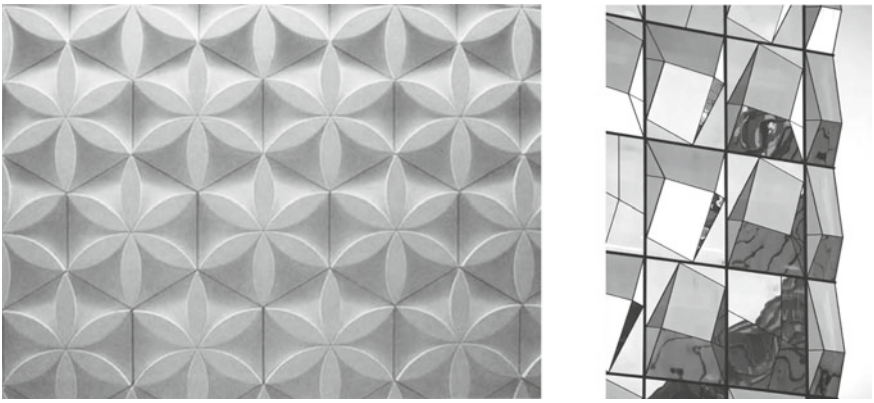


Fig. 3. The tile of the Stella retail store in Manhattan and the glass facade of the Turtec Tower in Seoul

selecting different forms of unit bars as symmetrical elements for translational symmetry operation. P2 space group operating principle, for example, select the corresponding parallelogram lattice, and then place the symmetry element as bar in the crystal lattice, based on planar point group operating principle of the rotation, mirror image operation generates epidermis local structure unit, finally, based on the operation principle of space group p2 translation and translation operation such as sliding reflect an entire construction epidermis keel structure, Finally, the epidermal filler was inserted with other constraints (Fig. 4). The generated epidermis keel rod has fewer types and rich Mosaic forms, which is convenient for industrial production and assembly of building epidermis.

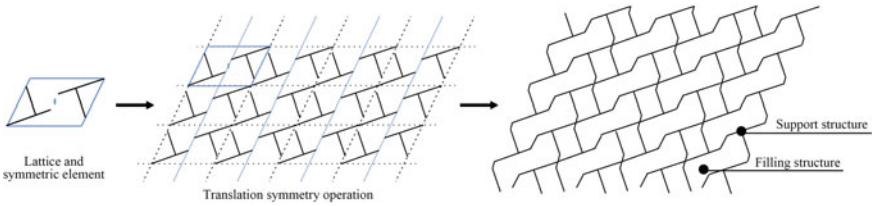


Fig. 4. Skin generation logic for translational symmetry operation of structural bars as symmetrical elements

The skin patterns generated by the p2 operation principle of plane space show the law of periodic Mosaic, and the operation principle of crystal group explains the mathematical basis of previous skin Mosaic research. Compared with the periodic Mosaic of regular polygons of building skin, the operation of building skin based on the principle of translational symmetry of planar crystal group can produce richer Mosaic combination forms.

4 Intervention of Interference Algorithm

Based on the above research on planar crystal group operation, the building skin prototype generated based on crystal group operation principle is both mathematical logic and external artistic, and is an ideal algorithm input parameter. Therefore, the geometric algorithm closely related to the principle of crystal group operation can be introduced into the architectural skin design method based on the principle of crystal group operation to pursue more diversified architectural skin pattern forms. The advantage of geometric algorithm intervention lies in that it can break the limitation of human brain and get uncertain or even infinite output results through limited input steps. The intervention of set algorithm can break the limitations and order of the building skin formed by the operation principle of crystal group and find a better solution from the essence of the problem. Algorithm intervention in the pursuit of complex forms of skin does not interfere with the logic of building skin generation. Complex geometric forms of skin are derived from the strategy of plane group generation. Similarly, complex geometric algorithms can produce intuitive skin pattern forms that are both rhythmic and artistic. Taking the interference algorithm as an example, the skin parameterization design method is discussed in combination with the skin generation algorithm based on the principle of plane group operation.

In the field of architectural design, interference algorithms can be used to intervene the generation of architectural form, space and skin. Intervention will interfere with the algorithm to the prototype design of generated on the basis of the principle of crystal group operating skin, break through the crystal group operating logic into epidermis both the limitations of design itself and stability, and tend to pursue to follow the inherent nonlinear aesthetic characteristics on the basis of mathematical logic to generate order, the pursuit of crystal group of logic operation supported by nonlinear changes, The established skin nonlinear operation under crystal group operation logic is carried out. By inputting a set of parametric variables that produce regular changes under the control of specified points, lines or patterns, the interference algorithm can regulate the skin elements at the macro level and output the skin variation forms under the control of several specified factors. Based on planar crystal group of the operating principle of choosing different forms of the lattice grid as a unit, under different interference sources intervention, control skin partial or whole follow certain regularity of the change, break of design on the basis of the principle of planar crystal group of the stability of the construction epidermis and regularity, output both external artistic form and inherent logic of the skin.

Point interference algorithm is to use one or several points to control a group of parameter variables, and with the interference points as the center, to produce interference deformation in the form of noise diffusion in the lattice skin unit for the preformed skin based on planar crystal group operation. The lattice skin element deformation operation is divided into two steps. Based on the interference parameters, the edge of the element is changed symmetrically to the center of the element, and then the deformed element is enlarged or reduced as a whole to obtain the final overall skin deformation effect. The deformation degree is centered on the interference point and gradually diffuses and changes gradually. Through the computer platform, the algorithm matrix can be realized on the building skin. Line interference algorithm uses one or several lines to control

multiple groups of parameter variables, which can also be understood as a series of points on the line under the control of the set. The skin change under the control of the line interference algorithm is the interference effect of the line centered outward diffusion gradual change of the skin element (Fig. 5). Image interference algorithm can extract the gray scale of the image as a parameter variable to determine the degree of interference to the element deformation. The gray scale data makes the morphological changes of the building skin more random and natural, and can present the artistic state of three-dimensional fluctuation on the two-dimensional building skin.

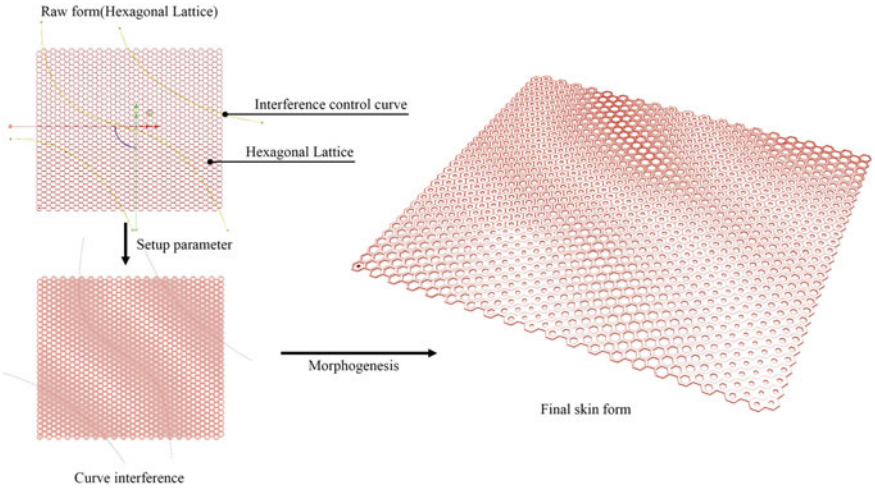


Fig. 5. Schematic diagram of curve interference algorithm for skin design

5 Conclusion

Based on the symmetry operation principle of planar crystal group, a new parametric design method of building skin based on mathematical basis is discussed by means of method explanation and examples. The generated building skin has both internal mathematical logic and external artistic quality. Selection based on planar crystal group of 5 kinds of planar lattice, 10 kinds of planar point group and 17 kinds of planar space group operation principle, this paper explains its operation principle in building skin design contains the inherent logic and diversity, demonstrated combined with various design elements applied in the field, the feasibility of building skin design. Through the disassembly of planar crystal group symmetry operation process, find the way to combine with the problems and elements in the design of architectural skin, transform the crystallographic symmetry group operation matrix into parameterized operation steps through programming language for visual operation, and generate skin prototype. Then the skin prototype is operated by geometric algorithm to generate the skin form with high complexity and leap dimension, and a design method of building skin generation based on planar crystal group is constructed.

The parametric design method of building skin based on the operation principle of planar crystal group has great research potential and space. It has the following characteristics: it fits with the process of building industrialization and digitalization; Generate rich skin effects with mathematical logic; Feasibility of auxiliary design; Feasibility of generation process; Complexity of generating results; Constructability of generative modes. In the stage of skin design, rich skin prototype forms can be generated by using the translational symmetry of planar crystal group, and further interference operation of parameters and variables can be carried out by intervening geometric algorithm. Based on the operation of mathematical logic, the external artistic expression of skin can be enriched, and rich architectural skin forms can be generated. At the same time, the building physical environment and skin structure can be optimized by combining performance simulation. In the skin construction phase, the symmetrical operation of the abstract planar crystal group can be used to deepen the skin entity, so that the construction of the building skin is compatible with the industrial prefabrication. In the future, the application of the operation principle of two-dimensional or even three-dimensional crystal group in architectural skin and space needs more research and attempts, which is also the focus of future exploration.

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