A Parametric Wave Joint for Robotic Fabrication of Digital Stereotomy

Wei Ye[1](✉), Yingzhou Gao[1], and Weiguo Xu[2]

[1] Tsinghua Shenzhen International Graduate School, Shenzhen 518055, China
yew21@mails.tsinghua.edu.cn

[2] School of Architecture, Tsinghua University, Beijing 100084, China

Abstract. This paper explores the potential of digital stereotomy in combination with robotic fabrication to increase the precision and complexity of stone processing. To enable the application of these techniques in outdoor environments, modular joints designed for robotic assembly are necessary. Additionally, the cutting process must be efficient and minimize material waste. To address these challenges, this research proposes a parametric wave joint design that enables rapid cutting and straightforward assembly by a robotic system. The joint contains motion space allowing it to slide into accurate assembly position, enabling the robot to complete the assembly without requiring highly precise vision or gripper in outdoor situations. Furthermore, the wave joint design eliminates the need for milling, reducing the processing time. The paper presents a robotic arm-cutting method for this joint and conducts experiments using foam and robotic arm hot-wire cutting to simulate stone cutting. The feasibility of the joint is tested through the assembly of a bent column, and finite element analysis is used to compare the stresses on two joint parts under shear force with different control parameters. The study confirms the feasibility of the wave joint design for robotic assembly and the efficiency of robotic arm cutting. The findings may inform the development of modular assemblies for robotic systems in stone processing applications.

Keywords: Digital stereotomy · Robotic fabrication · Wave joint · 3D assembly · Computational design

1 Introduction

Stereotomy, an ancient building construction technique, has been employed for centuries, relying mainly on manual processes prior to the emergence of computer technology [3]. Digital stereotomy refers to the use of digital technologies to plan and execute the cutting of stone blocks for use in building construction. Digital stereotomy has evolved from traditional stereotomy, which involves the manual measurement and cutting of stone blocks, and has been aided by the advancement of digital technology in recent years.

While previous studies have investigated various aspects of digital stereotomy, there are still challenges that need to be addressed to enable the practical application of these techniques in outdoor environments. Modular joints designed for robotic assembly are

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necessary, and the cutting process must be efficient and minimize material waste. To address these challenges, this research proposes a parametric wave joint design that enables rapid cutting and straightforward assembly by a robotic system.

This study adopts a design science methodology to devise a suitable joint design for digital stereotomy. The methodology involves an extensive review of the existing literature and practical applications of digital stereotomy to identify research gaps in the current theory. Based on the identified gaps, a novel solution is proposed, and a joint design is developed alongside a parametric generative algorithm. To evaluate the feasibility of the proposed design, robotic fabrication and finite element analysis are conducted.

The joint contains motion space allowing it to slide into accurate assembly position, enabling the robot to complete the assembly without requiring highly precise vision or gripper in outdoor situations. Furthermore, the wave joint design eliminates the need for milling, reducing the processing time. The paper presents a robotic arm-cutting method for this joint and conducts experiments using foam and robotic arm hot-wire cutting to simulate stone cutting. The feasibility of the joint is tested through the assembly of a bent column, and finite element analysis is used to compare the stresses on two joint parts under shear force with different control parameters. The study confirms the feasibility of the wave joint design for robotic assembly and the efficiency of robotic arm cutting. The findings may inform the development of modular assemblies for robotic systems in stone processing applications.

2 Systematic Literature Review

This paper presents a thorough literature review of the research of modular joints, categorizing joint forms into two main groups: 2D joint and 3D joint. The 2D joint refers to joints that can be formed by the same section line extrude. Conversely, the 3D joint refers to joints that cannot be formed by a single section line extrude.

The 2D joints can be further subdivided based on their section line of curves and polylines, such as dovetail suture tab design [6] and registration groove [1], where the authors use curves to form interlocking joints and grooves. Others, such as semi-circular masonry arches [3] and helicoidal skewed arch [9, 10], use polyline joints to increase friction and squeezing forces in the same direction to achieve a steady state. The Finger joint [7], proposed in 2015, has been used in the construction of the entity to confirm the validity of the conclusion. However, 2D joints cannot provide sufficient sliding resistance for a building's lateral forces in multiple directions.

On the other hand, 3D joints, due to their different section lines in each section, appear to resist sliding in cross-section directions. This paper further categorizes these joints into multi-groove joint, wave joint, and others. For example, to create interlocking conditions, drum face [1] and groove joint [8] are often cut to form multiple grooves that resist sliding. Moreover, the most common form of study used in digital stereotomy is the wave joint, which can be cut in a single pass using a wire saw, increasing work efficiency. Examples include wave-jointed blocks [12], cone joints [13], wave joints in catenary arch [5], and osteomorphic blocks [4]. Except for those mentioned above, a universal joint [11] will build a stable three-dimensional space using robotic arms.

These different types of joints provide viable construction methods, cross-section shapes, experimental methods, and analysis methods for digital stereotomy and modular blocks. From the perspective of the fabrication system, the 2D joints can all be cut by a wire saw but cannot provide sufficient sliding resistance. The 3D multi-groove joints provide enough sliding resistance, but cannot be cut by a wire saw. The 3D wave joints can be cut by a wire saw, but the cross-section contains some weak parts that are easy to damage.

Therefore, it is important to investigate the possibilities of combining the benefits of both 2D and 3D joints, such as creating a joint that has interlocking grooves or multi-grooves that can resist sliding in all directions while being cut by a wire saw. Furthermore, the joint design should consider the strength and durability of the joint in addition to the ease of assembly, making it a reliable and efficient method for digital stereotomy (Table 1).

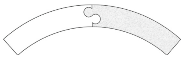
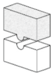
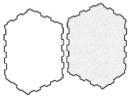
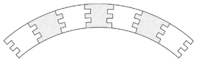
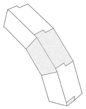
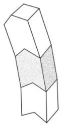
3 Design Development

This paper seeks to explore how a complex joint can be designed to combine sliding resistance on all sides, provide motion flexibility for easy assembly, and have wire saw cutting capabilities. Creating concave and convex joints from the module itself rather than adding new material prevents the deformation of different materials from impacting durability [12]. Interlocking joints are used to limit displacement. For a groove joint with only one groove, movement is possible along the direction of the groove. Dovetail joints can only move along the direction of their groove, as the successive contact surfaces of the groove form acute angles two by two (Fig. 1a). When the successive contact surfaces of the groove form right angles two by two, the parts can move along the groove and in the direction perpendicular to the groove (Fig. 1b). When the successive contact surfaces of the groove form obtuse angles two by two, the parts can move along the range of the combined angles of the inclined angles of the two inclined contact surfaces (Fig. 1c). The described movement along the inclined contact surfaces is discussed as motion space.

3.1 Motion Space

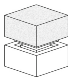
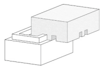
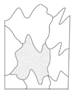
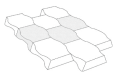
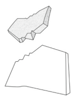
The motion space discussed in this paper refers to the motion space between two interlocking joint parts, which facilitates easy assembly while allowing for tolerable misalignment. The motion space of the third joint (Fig. 1c) allows misalignment within a certain angle and distance (Fig. 2a). However, the motion space is also making the assembly of the third type of joint easily disintegrate, by contrast to the second type of joint (Fig. 1b) whose sliding friction surfaces are parallel to each other, allowing disassembly force direction of only along the groove and perpendicular to the groove. In order to guarantee both motion space and interlocking, it is necessary to combine the two types of joints (Fig. 1b, c). The motion space is essential for the assembly process but has a negative impact on stability. To counteract this, the lower part of the groove joint is set as inclined surfaces and the upper part as parallel surfaces (Fig. 2b). To increase the motion space while still providing parallel surfaces for interlocking, the inclined surfaces are altered to smooth curved surfaces that connect to the parallel surfaces (Fig. 2c). Such

Table 1. Existing joint forms

Type	Interlocking forms	Applications	Processing	Authors
2D				
Curve		Arch	Software simulation	Melissa M. Gibbons et al. [6]
		Arch	3D printing	Nabila Afif et al. [1]
Line		Plate structures	CNC milling	Jan Knippers et al. 2015
		Semi-circular arch	Software simulation	C. Casapulla et al. [3]
		Arch	Software simulation	Elham Mousavian et al. [9]
		Space structure	Software simulation	Elham Mousavian et al. [9]

(*continued*)

Table 1. (*continued*)

Type	Interlocking forms	Applications	Processing	Authors
3D				
Multi-Groove joint		Arch	3D printing	Nabila Afif et al. [1]
		Stack assembly	3D printing	Baolian Liu et al. [8]
Wave joint		Arch	Wire saw cutting	Simon Weir et al. (2021)
		Modular assembly	3D printing	Ziqi Wang et al. [13]
		Arch	Wire saw cutting	Shayani Fernando et al. (2019)
		Space structure	3D printing	Niloufar Emami et al. (2020)
Others		Space structure	Wire saw cutting	Shayani Fernando et al. [11]

a junction allows for alignment errors during installation, as the two blocks can be slid into the correct installation position through the motion space between the joints, but after installation, the parallel faces prevent dissembling. Reduce stress concentrations This paper defines this type of joint to be a ⋂ type of joint.

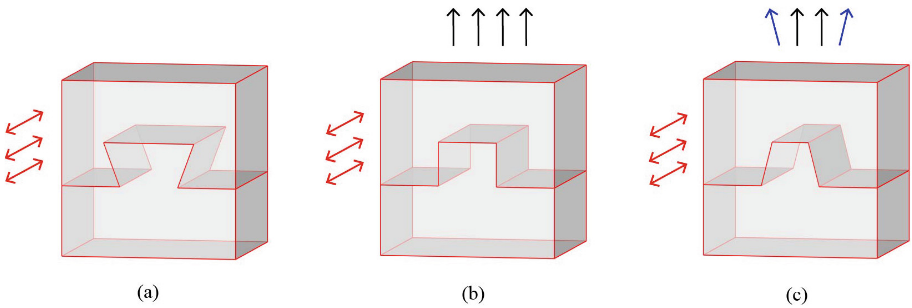


Fig. 1. The moveable direction of assembly with different joint shapes

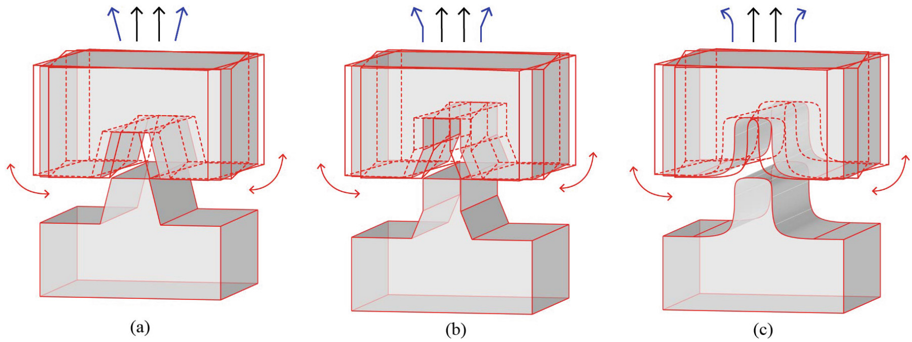


Fig. 2. Misalignment that can be accommodated by joints with motion space before assembly

3.2 Combination of ∧ Shape and ⋂ Shape

⋂ shaped joints with grooves in only one direction can be cut out by a wire saw, but such joints have the least resistance to displacement in the extended groove direction (Fig. 3a). ⋂ shaped joints with two non-intersecting grooves in different directions can also be cut out by a wire saw. Though such joints can resist horizontal displacement to a certain extent, the joint material needs better shear resistance (Fig. 3b). ⋂ shaped joints resist horizontal displacement best when the two grooves are perpendicular to each other and intersect, but cannot be cut out by wire saws, only by milling (Fig. 3c).

To address the low shear and tensile resistances in joints, Mousavian et al. studied interlocking joint shaping ∧ and ∨ [10]. The same two ∧ joints provide sliding resistance when they cross each other vertically and can also achieve wire saw cutting (Fig. 4a).

However, in order to ensure that wire saw cutting is possible, the centroid positions of the nodes are actually not interlocked.

Another example of a joint that provides sliding resistance and can be cut by a wire saw is the wave joint developed by Weir et al. (2015), see Fig. 4b. To ensure that the joint can be formed by direct cutting with a wire saw, the profile of the joint is always formed by a straight line. The profile shows a large degree of interlocking in the area near the outside of the joints, while there is almost no interlocking in the area near the center of the joints. This also makes the joints vulnerable to damage [5].

To allow joints to be processed directly by the wire saw, we chose to vertically intersect a ∧ shaped joint with a ⋂ shaped joint, with the ∧ shaped joint providing one side of the sliding resistance while the ⋂ shaped joint prevents displacement from the other side (Fig. 4c). For the objective of cutting only by wire saw, this paper forms a new wave joint based on the combined joint (Fig. 4c).

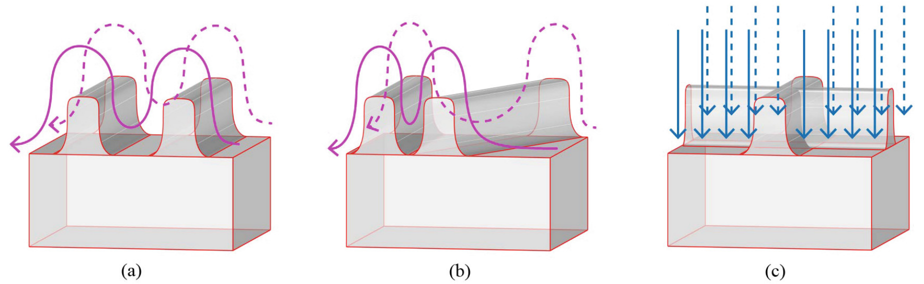


Fig. 3. The combination joint of two ⋂ shaped grooves and processing method

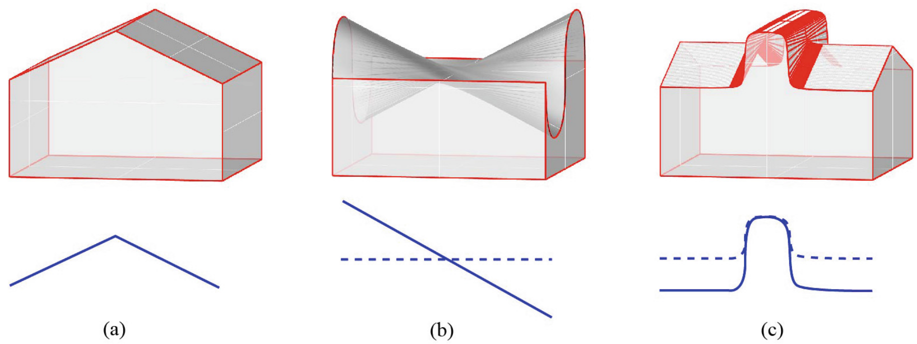


Fig. 4. Comparison of sections with different joints

4 Parametric Modeling

In order to automate and standardize the wave joint design process, the edge curves of the generated joints need to be controlled in a parametric way. The design of this joint needs to consider different parameters such as the height of the ⋂ part and the ∧

part, size and curvature of the motion space. The use of the joint also needs to compare the performance of joints generated with different control variables for a given identical parameter and find the optimal joint shape to use in different situations. In addition, since the poly-surface of this wave joint is generated by lofting the edge curves, the points on the curves need to be able to correspond one to the other, so a parametric calculation method is needed to calculate the curves.

4.1 Wave Curves Calculation

Given two parts Pi and Pj, with endpoints p1 and p2 and a middle point between endpoints p0. The purpose of the wave joint is to enable wire saw processing. This requires that the profile of the cutting surface consists of two corresponding points connected by a line. In order to better make the points of the cutting surface correspond to each other, a formula for generating joint edge curves is developed in this paper. The side curve is calculated as follows:

$$y = k\arctan(-|nx| + a) + \frac{k}{2}\pi, \# \quad (1)$$

When $x = 0$, x is located at the position p0. k is the occlusion depth constant, and a is the occlusion width constant. (k, $> a$ 0).

For the middle curve, while the width and vertex position of the $\bigcap$ part are the same as the side curve, the $\bigcap$ part of the middle curve is shorter than the side curve due to the $\wedge$ part needing in the other direction. So the constant b controlling the height of the $\wedge$ part is added to the calculation as follows:

$$y = b\arctan(-|nx| + 0.9a) + (k - b)\arctan(0.9a) + \frac{k}{2}\pi, \# \quad (2)$$

When $x = 0$, x is located at the position p0. b is the occlusion depth constant, and n, a is the occlusion width constant, $(k - b)$ is the height constant of the $\wedge$ shape ($0 < b$ $<k$). Figure 5a shows the guide curves of the joint (setting $k = 30$, $a = 20$, $b = 12$, $n = 1$). The red line is the side curve of the joint, the green line is the middle curve of the joint. The poly-surface of the wave joint is obtained by lofting the side curves and the middle curve.

When extra space between the two parts is needed, for example, it is necessary to leave space for adhesive. The curvature of the corner can be increased by lowering the constant n. As n is lowered, the constants k, b, a, should be adjusted accordingly so that the concave and convex parts can be nested and occluded. For the concave part constants k_1, n_1, a_1, b_1, when k_1, b_1= 1.05 k, b, n_1= 0.5 n, a_1 = 0.55 a, the side curve of the concave and convex parts are plotted as in Fig. 5b, where the blue line is the concave part and the red line is the convex part. The side curve and middle curve of the concave part are shown in Fig. 5c, the orange line is the middle curve. And Fig. 5d shows the middle curve of the concave and convex parts.

By adjusting different parameters, different joint shapes and sizes can be generated, and the joints generated by adjusting different control constants are shown in Fig. 6.

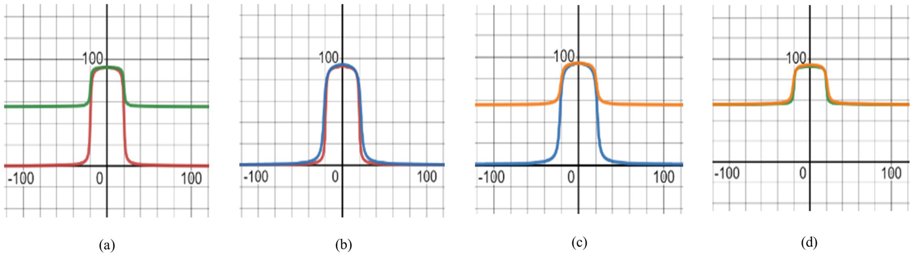


Fig. 5. Visualization of the equation of the edge curve of a concave part and a convex part

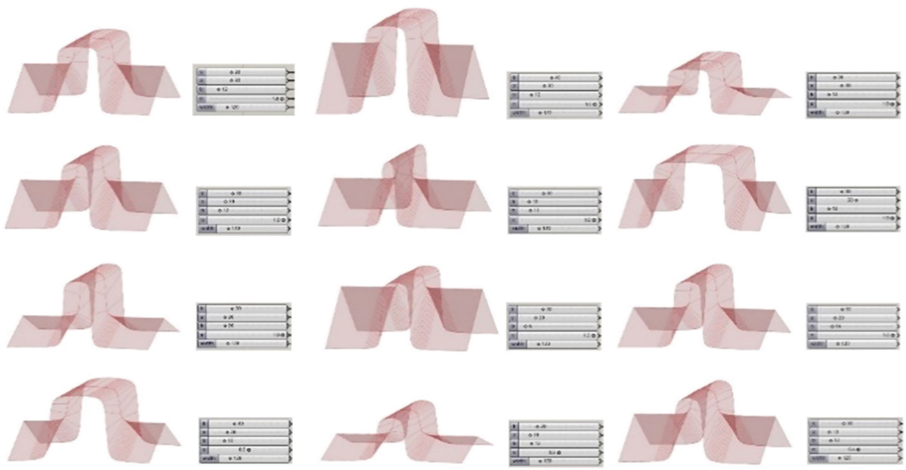


Fig. 6. Different joint shapes obtained by adjusting different constants in grasshopper. Joint shape (left) and its control constants (right)

4.2 Robotic Arm Cutting

The method proposed in this paper is a fabrication-oriented design. The reason why robotic arm stone processing is not yet widely used in construction scenarios is that robotic arm milling takes a lot of time and is too inefficient. However, the speed of robotic wire saw cutting is much faster compared to milling. For this reason, it is particularly important to design complex forms for the wire saw cut finish. The design of the wire saw cutting pattern should consider that the pattern consists of several ruled surfaces and that the integrity of the wire saw means that the path of the ruled surfaces should not be incorrectly damaged to the parts that do not need to be cut. It is also necessary to consider the shape of the tool head to avoid the wrong collision of the tool head with the object. The wave joint proposed in this paper can be done directly by wire saw cutting. Both the concave part and the convex part in this joint are cut twice by the robotic arm hot wire to be completed.

For the convex part, the guide curves of the robot arm is the edge curves of the wave joint. The two side curves and the middle curve are cut twice, as shown in Fig. 7 (left), the green lines are cut first, and then the red lines.

For the concave block, the guide curves cut by the robot arm is the inverted ∧ and ⋂ line. As shown in Fig. 7 (right), the green lines is cut first, and then the red lines.

Fig. 7. Simulation of hot wire cutting process in grasshopper

The prototyped was cut using a KUKA robotic arm with a hot wire tool. A bent column prototype was fabricated and assembled, see Fig. 8.

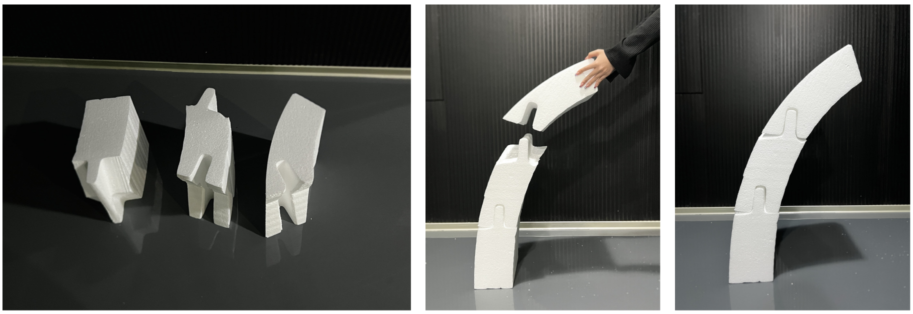


Fig. 8. A bent column cut with foam hot wire to simulate stone cutting

5 Future Study

This paper proposes a parametric wave joint from a processing and assembly point of view for digital stereotomy, presents its parametric generation method, and verifies its feasibility. Future research can experimentally test different parameters of these joints and verify which parameter can generate joint structures with performance that best matches the stone. In this paper, the performance of such joints with different ⋂ shape opening widths was simulated. Finite element analysis was done with Abaqus CAE for two different widths of wave joint with the same other parameters, as shown in

Fig. 9. From the simulation, it can be seen that when the ⋂ shape is narrower, the stress concentration is mainly located at the side of the ⋂ shape, while when the ⋂ shape is wider, the stress concentration is mainly located at the top of the ⋂ shape.

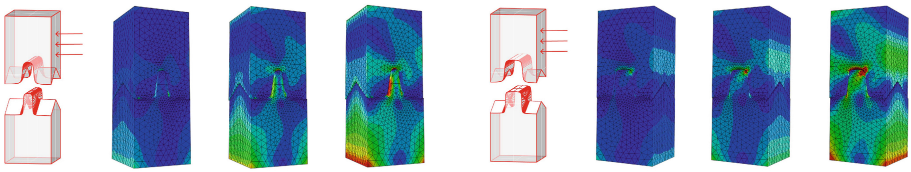


Fig. 9. The result of FEA on two different joints, narrow ⋂ shape (left), and wide ⋂ shape(right)

More finite element analysis experiments can be done in the future to further verify the feasibility of this kind of joint. In addition, in order for this parametric wave joint to be applied in real construction scenarios, its connection method should also be tested more, such as whether to add adhesive or waterproof material between the concave and convex parts, which will also bring about the change of parameters between concave and convex parts of the wave joint.

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