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Analytical Characterization on Pulse Propagation in a Semiconductor Optical Amplifier Based on Homotopy Analysis Method

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Abstract: Starting from the basic equations describing the evolution of the carriers and photons inside a semiconductor optical amplifier (SOA), the equation governing pulse propagation in the SOA is derived. By employing homotopy analysis method (HAM), a series solution for the output pulse by the SOA is obtained, which can effectively characterize the temporal features of the nonlinear process during the pulse propagation inside the SOA. Moreover, the analytical solution is compared with numerical simulations with a good agreement. The theoretical results will benefit the future analysis of other problems related to the pulse propagation in the SOA.

Keywords: Semiconductor optical amplifier; pulse propagation; homotopy analysis method; series solution

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1. Introduction

Semiconductor optical amplifier (SOA) is a key component in the optical information processing systems. Owning to advantages such as low cost, low power consumption, and fast optoelectronics response, it can be employed in the fiber sensor systems [1, 2]. In many SOA-based applications, pulse propagation is a basic problem and necessary to be deeply understood for the designers. In theory, lots of works have been reported on this issue. In [3], Agrawal *et al.* presented a comprehensive model for describing pulse propagation in an SOA and analyzed the self-phase modulation and spectral broadening of the output pulse. In [4], on the basis of the work of Agrawal *et al.*, Meccozi *et al.* developed an advanced model including the gain compression mechanism and obtained an approximate solution in the temporal domain by using the perturbation analysis. In [5], pulse propagation in a porlarization sensitive SOA was studied analytically. An approach similar to the way used in [6] was adopted, and an implicit solution in time domain was achieved. In [7], the authors constructed approximation analytical solutions for gain recovery dynamics experienced by a pulse propagated through an SOA, by making use of the multi-scale technique. In [8], through complete numerical simulation, the authors analyzed the rising and falling time of the output pulse by an SOA. In the past two decades, homotopy analysis method (HAM) has become more and more interesting in nonlinear scientific and engineering problems in different areas [9–12]. It is proven to be an effective method to solve nonlinear differential equations, especially those possessing strong nonlinearity.

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Owing to so many successful cases of solving nonlinear problems, it is reasonably expected that HAM will be a reliable technique for dealing with nonlinear models in optical systems.

In this paper, HAM is used to derive an explicit series solution for pulse propagation in SOAs. The remainder of the paper is organized as follows. In Section 2, the theory and basic equations characterizing the pulse propagation in an SOA are introduced and derived. Section 3 presents a detailed process of solving the equations in Section 2 by use of HAM. Section 4 offers necessary numerical experiments to validate the analytical results and some discussions. Conclusions are provided in Section 5.

2. Theory and basic equations

As described in [3], the equations governing the electrical field and the carrier inside the SOA can be expressed as follows without taking account for the transverse effect:

$$\frac{\partial A}{\partial z}+\frac{1}{v_g}\frac{\partial A}{\partial t}=\frac{1}{2}g(z,t)A(z,t)-\frac{1}{2}\alpha A(z,t) \quad (1)$$

$$\frac{\partial N(z,t)}{\partial t}=\frac{I}{eV}-\frac{N(z,t)}{\tau_e}-g(z,t)\frac{\lambda|A(z,t)|^2}{hc} \quad (2)$$

where A denotes the complex amplitude of the electrical field along the SOA, t is time, z is the distance measured from the left facet of the SOA, N is the carrier density, c is the light speed in vacuum, h is the Plank's constant, λ is the signal wavelength, $g=\Gamma a[N(z,t)-N_0]$ is the optical gain, and Γ is the mode confinement factor.

Let us introduce a transformation $\tau=t-z/v_g$ and rewrite A as $A=\sqrt{P}\exp(\mathrm{i}\phi)$, then we can get

$$\frac{\partial P}{\partial z}=(g-\alpha)P \quad (3)$$

$$\frac{\partial \phi}{\partial z}=-\frac{1}{2}\beta g \quad (4)$$

$$\frac{\partial g}{\partial \tau}=\frac{g_0-g}{\tau_e}-\frac{gP}{E_s} \quad (5)$$

where P is the normalized power of the pulse, ϕ is the phase of the pulse, β is the linewidth enhancement factor, g_0 is the small signal gain, and E_s is the saturation energy for input pulse.

Defining a function as follows:

$$h(\tau)=\int_0^L g(z,\tau)dz \quad (6)$$

then the solutions of (3) and (4) can be written as follows:

$$P_{\mathrm{out}}(\tau)=P_{\mathrm{in}}(\tau)\exp[h(\tau)-\alpha L] \quad (7)$$

$$\varphi_{\mathrm{out}}(\tau)=\varphi_{\mathrm{in}}(\tau)-\frac{1}{2}\beta h(\tau)\,. \quad (8)$$

If $h(\tau)$ is known, then the waveform and transient phase of the output pulse for a given input one are obtained.

Integrating (3) along the SOA length and eliminating gP by use of (5), we can get an ordinary differential equation that $h(\tau)$ obeys as follows:

$$\frac{dh}{d\tau}=\frac{g_0L-h}{\tau_e}-\frac{P_{\mathrm{in}}(\tau)}{E_s}[\exp(h)-1]\,. \quad (9)$$

Obviously, this is a nonlinear ordinary differential equation. For many existed techniques, it is unable to acquire the exact solution.

3. Analytical solution by HAM

In this section, we will employ HAM to derive approximate solution for (9). As seen from the original form of (9), the mathematical form of $P_{\mathrm{in}}(\tau)$ is unknown, and we substitute $P_{\mathrm{in}}(\tau)$ by a finite-order series, commonly the first several terms of its Taylor's series. This is feasible for any form of the input pulse, and thus the formation of (9) is completely knowable. Then we try to transform the nonlinear term in (9), $\exp(h)-1$, into a more tractable form. In this paper, we take the first three terms of its Taylor's series. It should be noted that after careful examination, the above mentioned approximation holds well in the typical parameter range for a typical SOA. Then (9) is translated into

$$\frac{dh}{d\tau}+a_1(\tau)h^3+a_2(\tau)h^2+a_3(\tau)h+a_4=0 \quad (10)$$

where

$$a_1(\tau)=\frac{b_1+b_2\tau^2+b_3\tau^4}{6E_s}$$
$$a_2(\tau)=\frac{b_1+b_2\tau^2+b_3\tau^4}{2E_s}$$
$$a_3(\tau)=\frac{b_1+b_2\tau^2+b_3\tau^4}{E_s}+\frac{1}{\tau_e}$$
$$a_4=-\frac{g_0L}{\tau_e}$$
$$P_{\text{in}}(\tau)\approx b_1+b_2\tau^2+b_3\tau^4 .$$

In the framework of HAM [9], we try to construct the zeroth-order deformation equation for (10)

$$(1-q)L[\Phi(\tau,q)-h_0(\tau)]=qc_0N[\Phi(\tau,q)] \qquad (11)$$

in which q is the embedded variable, $L[\Phi(\tau,q)]=\frac{\partial\Phi(\tau,q)}{\partial\tau}$ is the auxiliary linear operator, the nonlinear operator $N[\Phi(\tau,q)]=\frac{\partial\Phi(\tau,q)}{\partial\tau}+a_1(\tau)\Phi(\tau,q)^3+a_2(\tau)\Phi(\tau,q)^2+a_3(\tau)\Phi(\tau,q)+a_4$, c_0 is the homotopy parameter, used for controlling the convergence speed of the series solution. When $q=0$, (11) is transformed into the linear differential equation $L[\Phi(\tau,q)-h_0(\tau)]=0$, and the function $\Phi(\tau,q)$ is equal to the initial guess solution $h_0(\tau)$. When the embedded parameter q = 1, (11) is transformed into the original nonlinear differential (10), and the function $\Phi(\tau,q)$ is the solution of (10). As described by Liao's theory [9], the convergence region of the solution seems to increase as the homotopy parameter c_0 tends to zero.

Expanding $\Phi(\tau,q)$ to Taylor's series at $q=0$, we have

$$\Phi(\tau,q)=\sum_{m=0}^{\infty}h_m(\tau;c_0)q^m \qquad (12)$$

where $h_m(\tau;c_0)=\frac{1}{m!}\frac{\partial^m\Phi(\tau,q)}{\partial q^m}\bigg|_{q=0}$.

When $q=1$, the analytical solution of (10) can be expressed as

$$h(\tau)=\Phi(\tau,1)=\sum_{m=0}^{+\infty}h_m(\tau;c_0) \qquad (13)$$

with the initial condition $h(0;c_0)=H_0$.

Based on the zeroth-order deformation (11), we can derive the m-order deformation equation as described in [9]:

$$L[h_m(\tau)-\chi_m h_{m-1}(\tau)]=c_0R_m[h_{m-1}(\tau)] \qquad (14)$$

where $\chi_m=\begin{cases}0, m\le 1\\ 1, m>1\end{cases}$, and

$$R_m[h_{m-1}(\tau)]=\frac{1}{(m-1)!}\frac{\partial^{m-1}\Phi(\tau,q)}{\partial q^{m-1}}\bigg|_{q=0}=$$
$$h'_{m-1}+a_1(\tau)\sum_{i=0}^{m-1}h_{m-1-i}\sum_{j=0}^{i}h_{i-j}h_j+a_2(\tau)\sum_{i=0}^{m-1}h_ih_{m-1-i}+$$
$$a_3(\tau)h_{m-1}.$$

Equation (14) represents an infinite set of recurrence linear differential equations, which indicates that the original nonlinear equations are transformed into infinite linear equations. By using the symbolic computational software such as MATHEMATICA or MAPLE, the above equation can be solved iteratively, and thus the series solution of (10) can be achieved.

Observing the structure of the solutions of (14), $h_m(\tau)$ can be expressed in the following form:

$$h_m(\tau)=\sum_{n=0}^{7m+1}d_{m,n}\tau^n . \qquad (15)$$

Substitute (15) into (14), we have

$$\sum_{n=0}^{7m+1}d_{m,n}\tau^n=(1-c_0)\sum_{n=0}^{7m-6}d_{m-1,n}\tau^n+$$
$$c_0\int_0^{\tau}\left[a_1(t)\sum_{i=0}^{m-1}\left(\sum_{n=0}^{7(m-1-i)+1}d_{m-1-i,n}t^n\right)\cdot\sum_{j=0}^{i}\left(\sum_{n=0}^{7(i-j)+1}d_{i-j,n}t^n\cdot\sum_{n=0}^{7j+1}d_{j,n}t^n\right)+\right.$$
$$\left.a_2(t)\sum_{i=0}^{m-1}\left(\sum_{n=0}^{7i+1}d_{i,n}t^n\cdot\sum_{n=0}^{7(m-1-i)+1}d_{m-1-i,n}t^n\right)+a_3(t)\sum_{n=0}^{7(m-1)+1}d_{m-1,n}t^n\right]dt.$$
$$(16)$$

When (16) holds for an arbitrary τ, we have

$$d_{m,n} = (1-c_0)d_{m-1,n} + \frac{c_0}{n}\left[\left(\frac{b_1}{6E_s}A_{m-1,n-1} + \frac{b_1}{2E_s}B_{m-1,n-1} + \left(\frac{b_1}{E_s}+\frac{1}{\tau_e}\right)d_{m-1,n-1}\right) + \left(\frac{b_2}{6E_s}A_{m-1,n-3} + \frac{b_2}{2E_s}B_{m-1,n-3} + \frac{b_2}{2E_s}d_{m-1,n-3}\right) + \left(\frac{b_3}{6E_s}A_{m-1,n-5} + \frac{b_3}{2E_s}B_{m-1,n-5} + \frac{b_3}{E_s}d_{m-1,n-5}\right)\right] \quad (17)$$

where

$$A_{m-1,n-1} = \sum_{k=0,i=m-1-k}^{m-1} \sum_{j=0,l=n-1-j}^{n-1} d_{k,j}d_{i,l}d_{m-1-k-i,n-1-j-l}$$

$$B_{m-1,n-1} = \sum_{k=0}^{m-1}\sum_{i=0}^{n-1} d_{k,i}d_{m-1-k,n-1-i}\,.$$

Then the series solution of (10) can be expressed as

$$h = \sum_{m=0}^{\infty}\sum_{n=0}^{7m+1} d_{m,n}\tau^n \quad (18)$$

and the corresponding K-order approximation solution is as follows:

$$h(\tau) \approx \sum_{m=0}^{K}\sum_{n=0}^{7m+1} d_{m,n}\tau^n. \quad (19)$$

For the practical SOA, the approximated order K and the homotopy parameter c_0 can be properly selected by minimizing the residue of the original nonlinear equation [9].

By using the above approximation solution (19), we can get the output pulse profile and the frequency chirp as follows:

$$P_{\text{out}}(\tau) \approx P_{\text{in}}(\tau)\exp\left(\sum_{m=0}^{K}\sum_{n=0}^{7m+1} d_{m,n}\tau^n - \alpha L\right) \quad (20)$$

$$\Delta\upsilon_{\text{out}}(\tau) \approx \Delta\upsilon_{\text{in}}(\tau) + \frac{\beta}{4\pi}\left(\sum_{m=0}^{K}\sum_{n=0}^{7m+1}\frac{1}{n}d_{m,n}\tau^{n-1}\right). \quad (21)$$

4. Analysis and discussions

In this section, we will take some numerical experiments to verify our derivations of previous section. In the following, except for special statement, the parameters used in the simulations are taken the following values, $A_{\text{in}}=0.8\mu\text{m}^2$, $L=500\ \mu\text{m}$, $N_0 = 1.1\times10^{24}\text{m}^{-3}$, $a = 2.7\times10^{-20}\text{m}^2$, $\alpha = 1\,000\,\text{m}^{-1}$, $\beta = 4.8$, and $\Gamma = 0.06$. On the other hand, we take the approximate order K as 20 for the analytical solution by (20).

Figure 1 presents the relationship between the output profile and the input one. In this figure, the width of the input pulse is 20 ps, and the peak power is 50 mW. The solid line indicates the simulation results, and the dashed line denotes the analytical solution of (20). From the figure, it is obvious that our analytical solution is in good agreement with the numerical simulation. Moreover, this figure also indicates that our theoretical results are capable of describing the physical features during a pulse amplification process by an SOA.

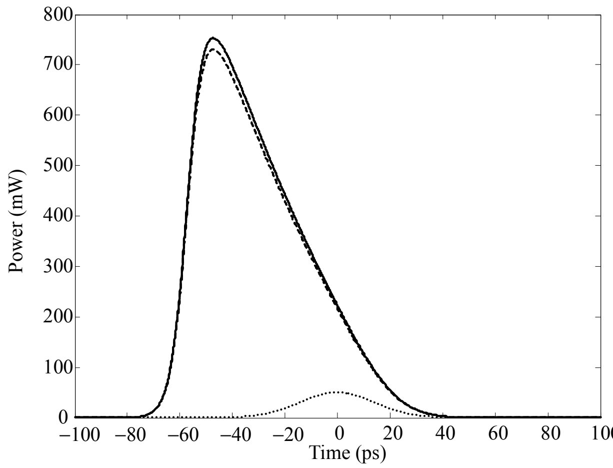

Fig. 1 Temporal profile of the input and output pulses: dotted line: the input pulse; dashed line: the analytical result by (20); the solid line: numerical result.

To further validate our research work, we examine more numerical examples taking large parameter range. Figure 2 displays the maximum relative error between the analytical result and the numerical one over a large parameter range. In this figure, it is clear that in a large parameter range, the correctness of our analytical result can be guaranteed. From a mathematical point of view, the findings of this paper can be regarded as a prolongation of the successful application of the HAM in the optical engineering domain. For the investigation into the optical signal processing by SOA, it is reasonable to take the HAM as a powerful tool in the research works.

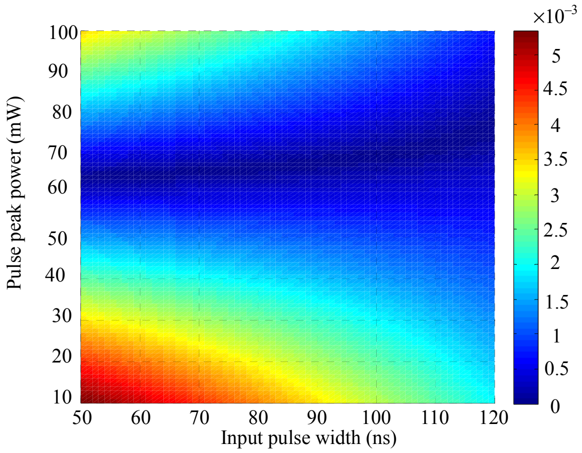

Fig. 2 Two-dimension map of the maximum relative error between the analytical result and the numerical one in the parameter space of the input pulse width and the peak power, and the bias current is 100 mA.

5. Conclusions

In this paper, the HAM is introduced into the investigation into pulse propagation inside an SOA. After some simplifications, the analytical expression of the output pulse is obtained. By comparing the numerical results, the HAM is proved to be an effective approach to research the dynamics in an SOA, and the correctness of the expressions is verified.

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